Singapore Mathematics and Algebraic Approach in Problem Solving

Efren S. Tellermo, Genesis G. Camarista

Abstract

This descriptive experimental research was conducted to determine the effect of Singapore Mathematics (SM) approach on the problem-solving ability of the fourth year students. Pre-test and post-test were utilized as instruments of this study. The means and standard deviations were computed for the descriptive analysis and the t-test for independent samples and t-test for dependent samples were used for inferential analyses. The findings showed that the students in both groups had a low performance in Math as indicated in the pre-test. Significant difference was noted in the post-treatment performance of the two groups, in favor of the experimental group. Significant differences were also noted between the pre- and post-treatment performance of each group. However, the experimental group has a better mean gain. With the aid of drawings, models or even real objects, students can see the visual representation of the problem's situation, leading to their better understanding and analysis of the problem. Mathematics teachers are encouraged to develop students' critical and logical thinking through problem-solving, to expose students to various strategies to make every Mathematics class lively and to elicit students' maximum participation.

Keywords: algebraic approach, problem solving, Singapore mathematics approach

Funded by the Department of Science and Technology – Science Education Institute

One of the basic and core subjects in the curriculum under the Department of Education in the Philippines is mathematics. Secondary teachers are expected that their students in mathematics are equipped with all the knowledge and skills specifically in problem solving. However many students have poor performance in mathematics, as has been reflected in the results of National Achievement Test.

Problem solving requires skills which prove to be difficult on the part of the students. It is considered a higher-order thinking process which requires higher order thinking skills. When a mathematical word problem is encountered, students tend to read and directly translate them into arithmetic operations. These often lead to misinterpretation of the word problem. Most verbal problems, although stated in real-life terms, seem to become abstract when it comes to the actual process of finding a solution. The usual way of solving problems in algebra is the algebraic approach or the equation method. Many students look at equation as very abstract and most often could not relate them to the statement of the problem. This is one reason why students have poor performance in problem solving. Another reason is their poor comprehension skills which lead to misinterpretation of the problem.

Students who are poor mathematical problem solvers lack self-regulation strategies that help successful students understand, analyze, solve, and evaluate problems. Lester (1994) distinguished good from poor problem solvers in five important aspects: (1) Good problem solvers know more than poor problem solvers and what they know; they know differently - their knowledge is well connected and composed of rich schemata; (2) Good problem solvers tend to focus their attention on structural features of the problem; poor problem solvers, on surface features; (3) Good problem solvers are more aware than poor problem solvers of their strengths and weaknesses; (4) Good problem solvers are better than poor problem solvers at monitoring and regulating their problem-solving efforts. (5) Good problem solvers tend to be more concerned than poor problem solvers about obtaining “elegant” solutions to the problems.

More sophisticated problem solvers transform word problems into object-based or mental models. Subsequent solutions are often qualitatively different because these models differentially support cognitive processing. A picture or diagram is often the most effective tool for discovery in explaining mathematical concepts as well as for solving problems. Instructions in diagram, drawing, and translating words into mathematical symbols indicate the effects toward better performance in problem solving (NCTM, 2000).

This study intends to investigate the effect of the Singapore Mathematics approach in the mathematical problem-solving ability of fourth year students. According to Brown (2013), Singapore Mathematics is a teaching method or the actual curriculum used for kindergarten through Grade Six in a small island country in the United States, which has become popular due to Singapore's consistent top ranking performance in an international assessment of student mathematics achievement called Trends in International Mathematics and Science Study (TIMSS). The Trends in International Mathematics and Science Study (TIMSS) is an international assessment of mathematics and science at the fourth and eighth grades that has been conducted every four years. From 1995 to 2011, Singapore has consistently been among the top performing countries.

Singapore Mathematics approach is a teaching method which has a consistent and strong emphasis on problem solving and model drawing. They use model drawings or manipulate real objects to solve and better understand word problems. Students are not just talking about concepts but seeing them through visual representation and hands-on activities. By using Singapore Mathematics models, students have a better understanding and analysis of problem situations.

The educational philosophies of John Dewey, Edgar Dale, and Jerome Bruner asserted that experience is essential to the learning process (Garrett, 1997) of the students. Dewey (in Garett, 1997) emphasized that the quality and the continuity of the experience are critical. Dale advanced these ideas by developing the Cone of Experience, a model that visually demonstrates how concrete experiences give meaning to abstract theories. According to Dale's research, the least effective method is at the top of the cone and involves learning from information presented through verbal symbols, i.e., listening to spoken words. Students often forget what they hear. The most effective methods at the bottom of the cone involve direct, purposeful learning experiences, such as hands-on or field experience. Direct purposeful experiences represent reality or the closest things to real, everyday life. Bruner's Theory of Instruction explains how learners move from inactive representations through iconic representations to symbolic representations in the learning process.

Methodology

To determine which instructional approach - Singapore Mathematics and algebraic approach - was more effective, the researcher administered pre-test and post-test before and after the experimental period. Experimental period lasts for eight weeks for both experimental and control groups. The experimental group was exposed to Singapore Mathematics approach and the control group, to algebraic approach.

Each group was handled by the same teacher and one hour was spent each day for each group. The same room was also used for classes throughout the experimental period. After four weeks, the schedule was interchanged.

The instrument used was pilot tested to determine its administrability and reliability. The expertise of a panel of jurors was sought to ascertain the content validity of the instrument. The reliability of the problem-solving test was determined through the computer processes factor using SPSS software. The final instrument containing 30 items had a reliability coefficient of 0.73 using Cronbach alpha.

For the descriptive statistics, mean and standard deviation were utilized. Inferential statistics used included the t-test for dependent samples and the t-test for independent samples, both set at 0.05 alpha level of significance.

Results and Discussions

Descriptive Data Analyses

The mean scores showed the levels of the participants' problem solving performance. Standard deviations were employed to determine the subjects' homogeneity and heterogeneity in the various areas of the investigation.

Pre-test scores in problem solving performance of the experimental and control groups. In general, the pre-treatment performance of fourth year students as a whole group and when classified as to sex was low. This low performance result might have been due to the fact that the students were not reviewed on the topics before they were given the pre-test.

Table 1

Pre-test Scores in Problem Solving Performance of the Experimental and Control Groups

Group	N	SD	Mean	Description
Experimental	40	3.38	7.90	Low
Male	22	3.81	7.59	Low
Female	18	2.82	8.28	Low
Control	40	4.15	7.90	Low
Male	22	3.80	7.41	Low
Female	18	4.58	8.50	Low

Note: 25 – 30=very high; 18 – 24=high; 13 – 17 =average; 7 – 12=low; 0 – 6=very low

Post-test scores in problem-solving performance of the experimental and control groups. Post-test scores revealed that the mean score obtained by the experimental group was high and that of the control group was average. Compared to the results of the pre-test, both groups had increased mean scores in the post-test; however, the experimental group had a better result. The standard deviation of the experimental group and the control group showed a wider dispersion from the means in the post-test as compared to that in the pre-test. This shows that students were more heterogeneous as regards their post-treatment problem- solving performance.

The high performance of the experimental group could be attributed to the students' interests in the strategy employed. Singapore mathematics models provide opportunities that arouse the interest of the students. Introducing problems through pictures will improve students' attitude towards problem solving. The problem-solving ability is enhanced if students visualize problems being solved.

Table 2

Post-test Scores in Problem-Solving Performance of the Experimental and Control Groups

Group	N	SD	Mean	Description
Experimental	40	5.14	18.93	High
Male	22	5.16	18.55	High
Female	18	5.23	19.39	High
Control	40	5.82	14.68	Average
Male	22	4.83	13.27	Average
Female	18	6.58	16.39	Average

Note: 25 – 30= very high; 18 – 24=high; 13 – 17=average; 7 – 12=low; 0 – 6=very low

Inferential Data Analyses

The significance of the differences in the level of performance of the fourth year students towards problem solving was determined in this study.

The t-test for independent samples set at 0.05 alpha was used to determine the significance of the differences among the participants classified as to treatment group. The t-test for dependent samples set at 0.05 alpha was also used to determine the significance of the differences in the pre-test and post-test mean scores of each group.

Difference in the pre-test scores in problem-solving performance between the experimental and the control groups. The results revealed no significant difference in the pre-treatment mathematics achievement between the students exposed to Singapore mathematics approach and those exposed to algebraic approach, t-test $t(78) = 0.000$, $p = 1.000$.

This result implies that both groups had more or less the same mathematical achievement at the start of the experimental period and were therefore comparable in terms of mathematics problem-solving performance.

Difference in the post-test scores in problem-solving performance between the experimental and control group. Post-test mean scores were again tested using t-test for independent samples set at .05 level to determine whether a significant difference will exist in the post-treatment mathematics achievement between the experimental group and the control group.

The results revealed that there was a significant difference in the post-treatment mathematics achievement between the students exposed to Singapore mathematics approach and those exposed to algebraic approach, $t(78) = 3.460$, p=0 .001.

This implies that the post-test mean score of the experimental group was significantly higher than that of the control group. This study confirms the report of National Council of Teachers of Mathematics (2000) that a picture or diagram is often the most effective tool for discovery in explaining mathematical concepts as well as in solving problems. Instruction using models has a greater effect towards better performance in solving problem. Pape (2004) added that more sophisticated problem solvers transform word problems into object-based or mental models. Subsequent solutions are often qualitatively different because these models support cognitive processing.

Difference in the pre-test and post-test in problem-solving performance of the experimental group. To ascertain whether a significant difference exists in the pre-test and post-test results, the researcher used the t-test for dependent samples set at .05 level of significance.

The t-test for dependent samples test revealed a significant difference in the pre- and post-treatment mathematics achievement of students exposed to Singapore Mathematics approach, $t(39) = 15.088$, p= 0 .000.

This implies that the experimental group's mean score in the post-test was significantly higher than that of the pre-test. Although the Singapore mathematics approach was new to the students, they were still able to adapt to and apply the strategy. Therefore, the results show that the Singapore Mathematics approach is effective and applicable especially to high school students.

Difference in the mean gains in problem-solving performance of the experimental and the control groups. To test whether a significant difference exists in the mean gains of the two treatment groups, the researcher utilized the t-test for independent sample.

Results revealed a significant difference in the mean gains of the experimental and control groups, $t(78) = 4.308$, p =0 .000.

The results indicate that although both groups gained, the experimental group had a greater mean gain and therefore had a better performance compared to the control group. Problem solving requires higher-order thinking skills. Thus, problems cannot be solved by students' memorization of solution

but by using critical and logical thinking. Successful problem solving is not possible without first representing the problem appropriately. Visualizing can help students better understand problem situations through the use of Singapore Mathematics models. Through Singapore mathematics approach, difficult problems were made easier. The essence of mathematics is not to make simple things complicated but to make complicated things simple.

Table 3

Mean Gain Scores in Problem-Solving Performance of the Experimental and Control Groups

Group	N	Pre-test Mean	Post-test Mean	Mean Gain
Experimental	40	7.90	18.93	11.03
Male	22	7.59	18.55	10.95
Female	18	8.28	19.39	11.11
Control	40	7.90	14.68	6.78
Male	22	7.41	13.27	5.86
Female	18	8.50	16.39	6.79

Conclusions

On the basis of the findings, the following conclusions were made:

The high performance of the students in the experimental group manifests the students' interest in the strategy employed. This suggests that by using Singapore mathematics approach, students become very comfortable with solving problems. The high performance may also be attributed to the fact that when students use Singapore Mathematics approach in solving mathematics problems, they can draw models that help them visualize abstract situations. With the aid of models, diagrams, or manipulative, it is be easier for teachers to help students' gain better understanding and analysis of the problem. By drawing models for a problem situation, the students unknowingly unlock the problem's difficulty. This is supported by NCTM (2000) which contends that a picture or diagram is often the most effective tool for discovery in explaining mathematical concepts as well as in solving problems.

Furthermore, the students exposed to Singapore Mathematics approach had greater mean gain. By employing Singapore Mathematics approach in solving mathematical problems, the students' performance improved. It helped them practice their creativity and tickled their artistic minds as they draw

models. On the other hand, the algebraic approach may also help improve students' mathematics problem-solving performance, but it may be used only when the lesson is mastered.

Implications

The findings of the present study have led to certain implications for theory and for practice in relation to the effectiveness of the Singapore mathematics approach and algebraic approach in the problem-solving performance of the fourth year students.

For theory

The result of this study showed that Singapore Mathematics approach is more effective than algebraic approach in solving mathematical problems. This is advanced by the experiential learning theory (ELT) of Kolb (1981). In Kolb's ELT, immediate or concrete experiences lead to observations and reflections. These reflections are then assimilated (absorbed and translated) into abstract concepts with implications for action which persons can actively test and experiment with and in turn enable the creation of new experiences. With the aid of drawings, students can see the visual representation of the problem's situation and lead to their better understanding and analysis of the problem.

However, both experimental and the control groups also increased significantly. This implies that both algebraic approach and Singapore Mathematics approach can be alternatively used in solving problems. This is affirmed by "Dale's Cone of Experience". Dale's argument was not that more concrete experiences were better than more abstract ones. Dale believed that any and all of the approaches could and should be used, depending on the needs of the learner. During the presentation of the lesson on problem solving, Singapore Mathematics can be introduced for a better grasp of the lesson. Then, students can use algebraic approach as long as they have mastered the lesson.

This study further showed that sex does not affect students' mathematics problem-solving performance. This contradicts with that of Fennema and Sherman's (1978) that sex differences is apparent in high school.

For Practice

The result implies that employing Singapore Mathematics approach as a strategy in solving mathematical problems is a better approach especially on whole numbers and decimals, fractions, percentage, ratio and proportion.

The ultimate goal of every teacher teaching problem solving in the classroom is to equip students with necessary skills and knowledge needed to solve every problem. Students should be able to learn with understanding and develop their critical thinking and problem-solving skills. This is attainable if teachers give focus on the process rather than on the result. Having students exposed to Singapore mathematics approach as a strategy in problem solving may likely lead to success. But before this can be achieved, teachers must be equipped first with such skills.

Teachers are the key factors in the mathematics teaching learning process. The success or failure of mathematics instruction depends on such factors. The experience that students gain through their interaction with the teacher greatly influences their construction of mathematical knowledge. To be able to teach problem solving effectively, teachers need to be problem solvers themselves.

As observed in the conduct of the study, correct solutions on certain problems depend on the students' comprehension of problem situations and the teacher's expertise in facilitating the process. The problems given to the students are ascending as to the degree of the problem's complexity. Fulfilling experiences and appreciation of a work well-done motivate the students and become more interested in attending classes.

When Singapore Mathematics approach, was first introduced, the students took a longer time solving problems since the strategy is new to them. But when students got used to this approach, they became more creative and organized in finding and showing solutions. However, in the control group, the fast learner becomes faster in arriving at the solution.

The result of this study on sex differences contradicts the findings of Fennema and Sherman (1978) that states sex differences are apparent in high school. Furthermore, this result may inform the female about their ability in problem solving.

Recommendations

In the light of the conclusions derived from the findings of this study, the following recommendations are advanced:

When it comes to mathematics performance, it is in problem solving where students most often get low performance. Mathematics teachers are encouraged to develop students' critical thinking through problem-solving. Students should be given opportunities to enhance and maximize their problem solving skills', in fact, mathematics instruction should be problem solving-based.

Teachers are encouraged to expose students to various strategies to make every mathematics class lively and to elicit students' maximum participation. One strategy recommended is the Singapore Mathematics approach. Since this approach requires a problem solver to draw models or even manipulate real objects, it also necessitates enough time and effort to visualize problem situations. Pace of instruction must be adjusted.

Schools should implement remedial reading programs to increase students' comprehension. Reading comprehension must be possessed by the students, so students can easily translate verbal phrases/sentences into mathematical models and eventually into mathematical phrases/sentences.

Textbooks should include routine and non-routine problems with corresponding appropriate strategies suggested to solve them. Singapore Mathematics approach should be integrated in these textbooks.

Similar studies on the use of Singapore Mathematics approach in solving mathematical problems should be conducted with longer duration.

References

Brown , L. (2013). *What's Singaporean math?* Retrieved 2 September 2013 from http://www.pbs.org/parents/education/math/math-tips-for-parents/whats-singapore-math/

Fennema, E. & Sherman, J. (1978).Sex-Related differences in mathematics achievement and related factors: A further study. *Journal of Research in Mathematics Education*, *9*(3), 189-203. DOI: 10.2307/748997

Garett, L. (1997), Dewey, Dale, & Bruner: Educational philosophy, experiential learning, and library school cataloging instruction. *Journal of Education for Library and Information Science*, *38*(2),. 29 – 136. DOI: 10.2307/40324216. Retrieved November 30, 2013 from http://www.jstor.org

Kolb, D.A. (1981). Experiential Learning Theory and The Learning Style Inventory: A Reply to Freedman and Stumpf. *The Academy of Management Review*, *6*(2), 289-296. Retrieved November 30, 2013 from http://www.jstor.org

Lester, F. K.(1994). Musings about problem-solving research. *Journal for Research in Mathematics Education*, *25*(6), 660-675.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.